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Higher Dimensional Bianchi Type –I Model Cosmic Strings Coupled with Perfect Fluid to in Bimetric Relativity

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Abstract

Restricting to a particular type of background metric it is found that there is no contribution from cosmic string coupled with perfect fluid to the higher 3 five dimensional, 4 six-dimensional and 5 n- dimensional Bianchi type –I cosmological model in bimetric relativity.

Keywords: Cosmic String, Perfect Fluid, Singularity, Bimetric Relativity, AMS SUB. CODE: 83C05 (General Relativity)

Introduction

Rosen[6] (1973) proposed the bimetric theory of relativity to remove some of the unsatisfactory features of the general theory of relativity in which there exist two metric tensor at each point of space-time gij, which describes gravitation and background metric γij , which enters into 2 the field equations and interacts with gij but does not interact directly with matter. One can regard γij as describing the geometry that exists if no matter were present.

Accordingly at each space-time point one has two line elements-

$$ds^2 = g_{ij}\, dx^i dx^j \quad (1.1)$$

And $$d\sigma^2 = \gamma_{ij}\, dx^i dx^j \quad (1.2)$$

where ds is the interval between two neighbouring events as measured by a clock and a measuring rod. The interval dσ is an abstract or a geometrical quantity not directly measurable, one can regard it as describing the geometry that exist if no matter were present.

In Rosen's bimetric theory of relativity, Mahurpawar [14, 15, 16] (2003-2008- 2001) has obtained nil contribution of cosmic strings in five, six and generalized n- dimensional static plane symmetric cosmological model. We extended work in Bianchi type- I as non-existence of Maxwell fields [17] (1997), cosmic strings coupled with Maxwell's fields [18] (2003) vacuum solutions [19] (2003-04) higher five and six dimensional axially symmetric [20, 21] (2004- 2004) in bimetric relativity. Rosen [7,8] (1975-1980), Yilmaz [4] (1975), Karade[12](1980), Reddy [2](1989), Mohanty [3](2002), Deo[10,11] (2004-2013), Adhav [5](2005) have studied several; aspects of bimetric theory of relativity.

The string theory was developed to describe an event at early stages of evolution of the universe. Cosmic string arises during phase transition after the big bang explosion as the temperature goes down below some critical temperature [Zel'dovitch [23] (1975), Kibble [13] (1976), Vilenkin [1] (1982), Latellier [9] (1983) has initiated the study of cosmic strings in general by solving the Einstein field equations and obtained various cosmological models with string dust sources.

In this paper we extended my work[22] (2014) and studied higher six dimensional and n-dimensional non-static Bianchi type-I cosmological models with cosmic strings coupled with perfect fluid distribution and observed that there are no contributions of cosmic strings coupled with perfect fluid in this model. Only vacuum models were found

The Field Equations

The field equations of Rosen's bimetric theory of relativity read as

$$N_{ij} - \frac{1}{2} N g_{ij} = -8\kappa T_{ij}, \quad (2.1)$$

where $N_j^i = \frac{1}{2}\gamma^{ab}\left(g^{hi} g_{hj|a}\right)_{|b}$

And $\kappa = \left(\frac{g}{\gamma}\right)^{1/2}$, $N = N_i^j$, $g = det(g_{ij})$, $\gamma = det(\gamma_{ij})$ (2.2)

Summing over a and b from 1 to 4, a vertical bar (l) denotes the covariant differentiation with respect to γ_{ij}.

Five-dimensional Bianchi type-I cosmological model for cosmic strings coupled with perfect fluid

We consider the non- static space-time described by the five dimensional Bianchi type-I mode

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 + D^2du^2 \quad (3.1)$$

where A,B,C and D are function of time "t" only.

The flat metric corresponding to (3.1) is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + du^2 \quad (3.2)$$

Signature of metric (3.1) is (-, +, +, +, +) i.e. +3

Assume that the space-time is filled with cosmic strings coupled with perfect fluid whose energy- moment tensor is given by-

$$T_i^j = T_{istring}^j + T_{iperfect\ fluid}^j \quad (3.3)$$

where $T_{is}^j = \rho v_i v^j$ -λ$x_i x^j$ (3.4)

where ρ is the rest energy density for the cloud of strings with particle attached along the extension thus, $\rho = \rho_p + \lambda$

where ρ_p is the particle energy density, λ is the tension density of the strings and v^i the four velocity for the cloud of particle, x^i the flow of the matter the four vector which represents string direction which is essentially the direction of anisotropy.

$$v_i v^j = -1 = -x_i x^j$$

and $v_i x_i = 0$

$$T_{ip}^j = (\varepsilon + p)v_i v^j + pg_i^j \quad (3.5)$$

where ε being the matter density, p is the pressure

The components of energy tensor for a strings cloud field equations are-

$$-\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(-\lambda + p) \quad (3.6)$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(p) \quad (3.7)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(p) \quad (3.8)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi\kappa(p) \quad (3.9)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + \left(\frac{D_4}{D}\right)_4 = 16\pi\kappa(\rho + \varepsilon) \quad (3.10)$$

where the suffix 4 follows an unknown function of A, B, C and D denotes ordinary differentiation with respect to time t and $\kappa = ABCD^2$

Equations (3.6) to (3.10) yields,

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{B_4}{B}\right)_4 = \left(\frac{C_4}{C}\right)_4 = \left(\frac{D_4}{D}\right)_4 \quad (3.11)$$

Using equation (3.11) in equations (3.6) and (3.10), we get

$$2\rho + 2\varepsilon + 4p - 2\lambda = 0 \quad (3.12)$$

In view of the reality conditions i.e.

$\rho > 0, \varepsilon > 0, p > 0, \lambda > 0$.

which shows that five- dimensional non-static Bianchi type- I model representing the cosmic strings coupled with perfect fluid does not exist in bimetric relativity.

when $\rho = 0, \varepsilon = 0, p = 0, \lambda = 0$ (vacuum) equations(3.6) to(3.10) admit the solution

$$A = B = C = D = e^{\omega t} \quad (3.13)$$

where ω is the constant of integration.

In view of equation (3.13), the line element (3.1) takes the form

$$ds^2 = -dt^2 + e^{2\omega t}(dx^2 + dy^2 + dz^2 + du^2) \quad (3.14)$$

It is interesting note that equation (3.14) is free from singularity and for ω=0 it reduces to flat one. This result is analogous to the result obtained by Mahurpawar [22].

Six-dimensional Bianchi type-I cosmological model for cosmic strings coupled with perfect fluid

We consider the non- static space-time described by the Six dimensional non-static Bianchi type-I mode

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 + D^2(du^2 + dv^2) \quad (4.1)$$

where A,B,C and D are function of time "t" only.

The flat metric corresponding to (3.1) is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + du^2 + dv^2 \quad (4.2)$$

Signature of metric (3.1) is (-, +, +, +, +, +) i.e. +4 Assume that the space-time is filled with cosmic strings coupled with perfect fluid whose energy- moment tensor is given by-

$$T_i^j = T_{istring}^j + T_{iperfect\ fluid}^j \quad (4.3)$$

where $T_{is}^j = \rho v_i v^j$ -λ$x_i x^j$ (4.4)

where ρ is the rest energy density for the cloud of strings with particle attached along the extension thus, $\rho = \rho_p + \lambda$

where ρ_p is the particle energy density, λ is the tension density of the strings and v^i the four velocity for the cloud of particle, x^i the flow of the matter the four vector which represents string direction which is essentially the direction of anisotropy.

$$v_i v^j = -1 = -x_i x^j$$

and $v_i x_i = 0$

$$T_{ip}^j = (\varepsilon + p)v_i v^j + pg_i^j \quad (4.5)$$

where ε being the matter density, p is the pressure

The components of energy tensor for a strings cloud field equations are-

$$-\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + 2\left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(-\lambda + p) \quad (4.6)$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + 2\left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(p) \quad (4.7)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 + 2\left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(p) \quad (4.8)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi\kappa(p) \quad (4.9)$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + 2\left(\frac{D_4}{D}\right)_4 = 16\pi\kappa(\rho + \varepsilon) \quad (4.10)$$

where the suffix 4 follows an unknown function of A, B, C and D denotes ordinary differentiation with respect to time t and $\kappa = ABCD^2$

Equations (3.6) to (3.10) yields,

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{B_4}{B}\right)_4 = \left(\frac{C_4}{C}\right)_4 = \left(\frac{D_4}{D}\right)_4 \quad (4.11)$$

Using equation (4.11) in equations (4.6) and (4.10), we get

$$3\rho + 3\varepsilon + 5p - 3\lambda = 0 \quad (4.12)$$

In view of the reality conditions i.e.

$\rho > 0, \varepsilon > 0, p > 0, \lambda > 0$.

which shows that that six dimensional non-static Bianchi type- I model representing the cosmic strings coupled with perfect fluid does not exist in biometric relativity.

when $\rho = 0, \varepsilon = 0, p = 0, \lambda = 0$ (vacuum) equations(4.6) to(4.10) admit the solution

$$A = B = C = D = e^{\omega t} \tag{4.13}$$

where ω is the constant of integration.

In view of equation (4.13), the line element (4.1) takes the form

$$ds^2 = -dt^2 + e^{2\omega t}(dx^2 + dy^2 + dz^2 + du^2 + dv^2) \tag{4.14}$$

It is interesting note that equation (4.14) is free from singularity and for ω=0 it reduces to flat one. This result is analogous to the result obtained by Mahurpawar [22] (2014).

n-dimensional Bianchi type-I cosmological model for cosmic strings coupled with perfect fluid

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + C^2dz^2 + D^2\sum_{u=1}^{u=n-4} du^2 \tag{5.1}$$

where A,B,C and D are the functions of time "t" only and $\kappa = ABCD^2$

The corresponding flat metric is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + \sum_{u=1}^{u=n-4} du^2 \tag{5.2}$$

Signature of metric (4.1) is + (n-2).

Assume that the space-time is filled with cosmic strings coupled with perfect fluid whose energy- moment tensor is given by-

$$T_i^j = T_{istring}^j + T_{iperfect\ \ fluid}^j \tag{5.3}$$

The field equations (2.1) for the metric (5.1) and (5.2) corresponding to the energy momentum tensor (5.3) can be written as-

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + (n-4)\left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(-\lambda + p) \tag{5.4}$$

$$\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + (n-4)\left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(p) \tag{5.5}$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 + (n-4)\left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(p) \tag{5.6}$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + (n-6)\left(\frac{D_4}{D}\right)_4 = -16\pi\kappa(p) \tag{5.7}$$

$$\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 + (n-4)\left(\frac{D_4}{D}\right)_4 = 16\pi\kappa(\rho + \varepsilon) \tag{5.8}$$

where the suffix 4 follows an unknown function of A, B, C and D denotes ordinary differentiation with respect to time t and $\kappa = ABCD^2$

Equations (4.4) to (4.8) yields,

$$\left(\frac{A_4}{A}\right)_4 = \left(\frac{B_4}{B}\right)_4 = \left(\frac{C_4}{C}\right)_4 = \left(\frac{D_4}{D}\right)_4 \tag{5.9}$$

Using equation (4.11) in equations (4.6) and (4.10), we get

$$(n-3)\rho + (n-3)\varepsilon + (n-1)p - (n-3)\lambda = 0 \tag{5.10}$$

In view of the reality conditions i.e. $\rho > 0, \varepsilon > 0, p > 0, \lambda > 0$.

which shows that that n- dimensional non-static Bianchi type- I model representing the cosmic strings coupled with perfect fluid does not exist in biometric relativity.

when $\rho = 0, \varepsilon = 0, p = 0, \lambda = 0$ (vacuum) equations(4.6) to(4.10) admit the solution

$$A = B = C = D = e^{\acute{\omega} t} \tag{5.11}$$

where ώ is the constant of integration.

In view of equation (5.11), the line element (5.1) takes the form

$$ds^2 = -dt^2 + e^{2\acute{\omega} t}(dx^2 + dy^2 + dz^2 + \sum_{u=1}^{u=n-4} du^2) \tag{5.12}$$

It is interesting note that equation (5.12) is free from singularity and for ώ=0 it reduces to flat one. This result is analogous to the result obtained by Mahurpawar [22](2014).

Conclusion

Here we studied higher five, six dimensional and n-dimensional non- static Bianchi type-I model in Rosen's Bimetric theory of relativity. It is observed that the matter field in above source does not survive and hence only vacuum models (3.14), (4.14) and (5.12) are obtained. The vacuum models (3.14),(4.14) and (5.12) so obtained is found to be free from singularity and reduces to be non-static confirmally flat when ω =0 and ώ=0. The conclusion arrive at viz., λ=0, ε=0, ρ=0, p=0 are invariant statement and hold in all co-ordinate system even though we have derived these in commoving system.

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